Cellular Automaton Traffic Flow Model

モデル化とシミュレーション特論
2021 年度前期
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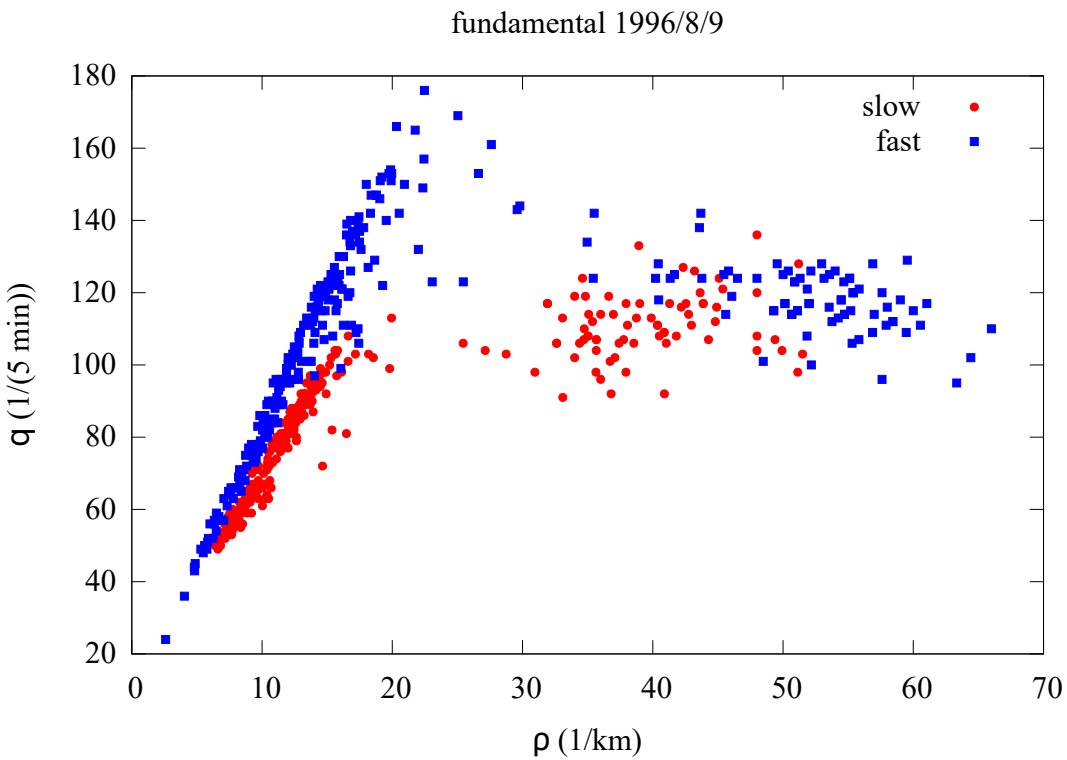
Fundamental Diagram: density-flow relations

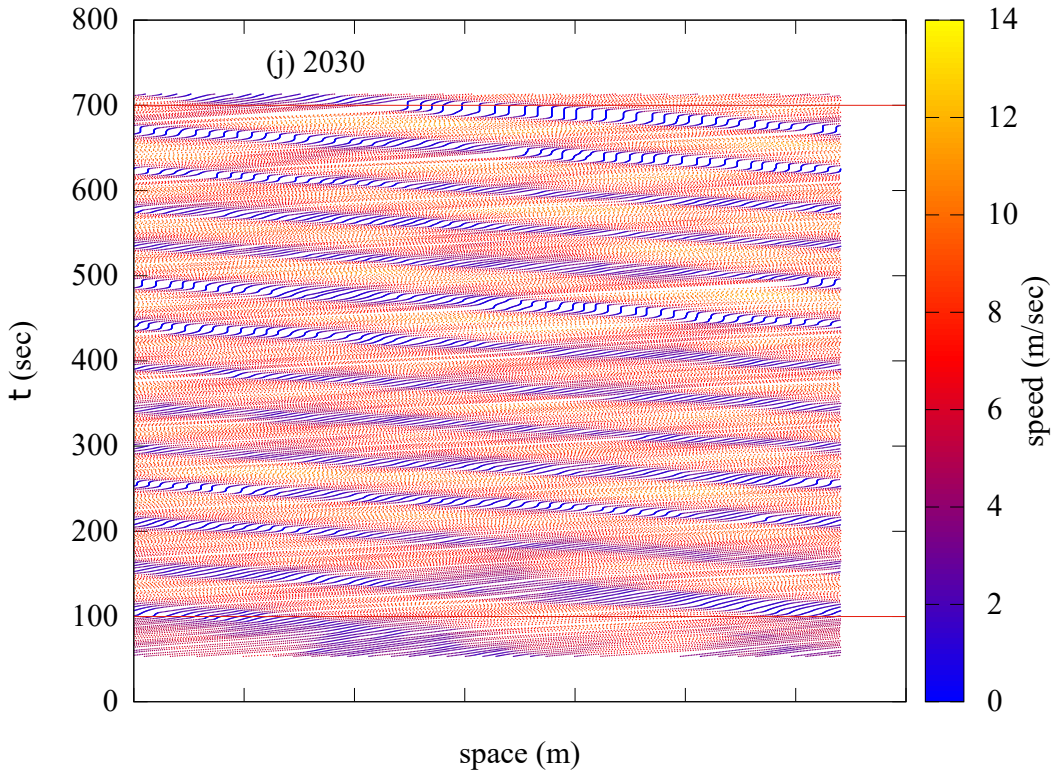
Data observed at Tomei Expressway



- Free and congested flows
- Broad distribution for congested flows

Spacetime Diagram

Data taken at Nagoya Dome Stadium



- Cars run rightward.
- Jam clusters propagate backward.

Discrete model of traffic flow

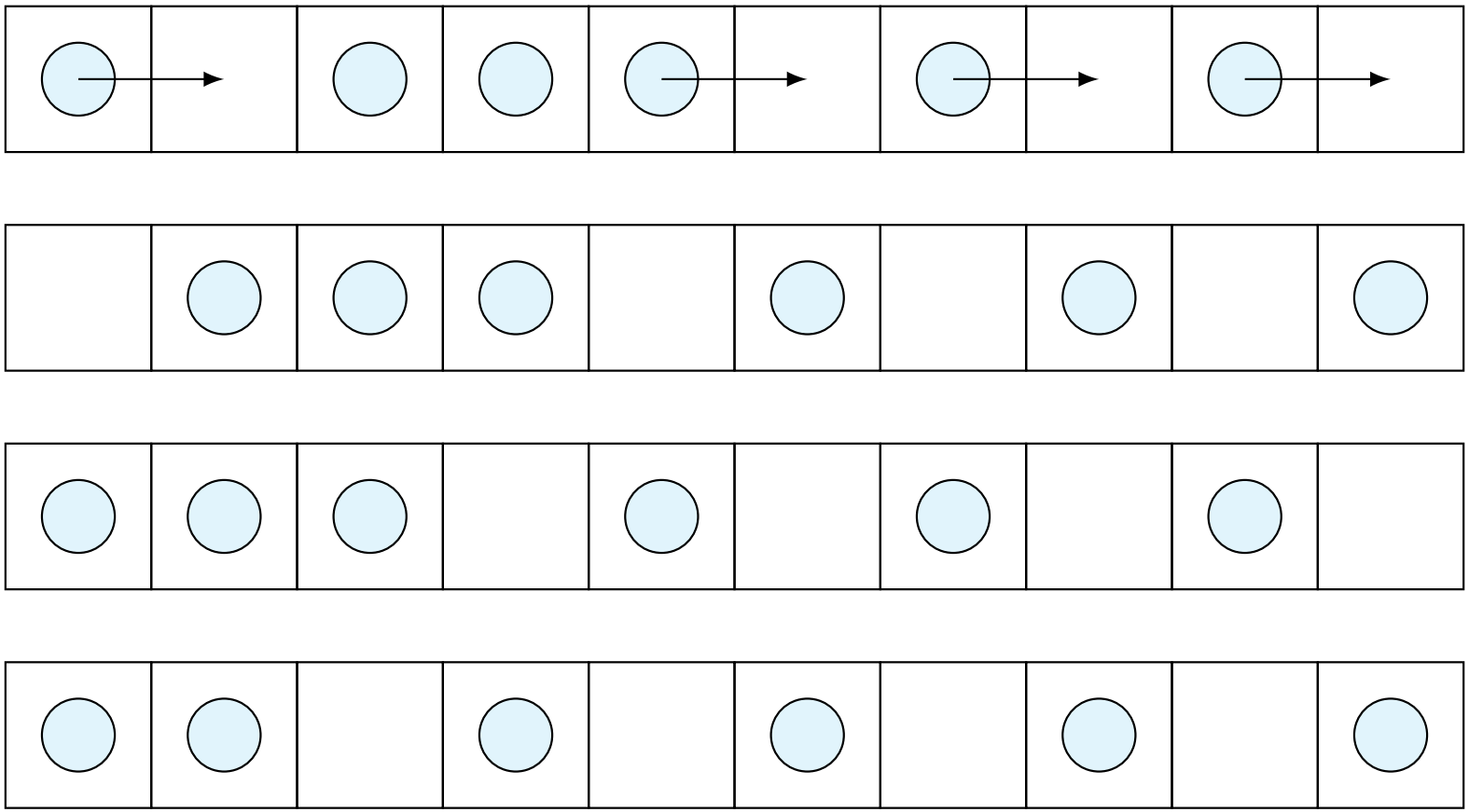
- Cars have finite length ($3 \sim 4$m for passenger cars)
- Any pair of cars can not occupy the same place (volume exclusion effect, **排除体積効果**)
- A car follows the motion of the preceding car with delay

Rule 184: Simplest model

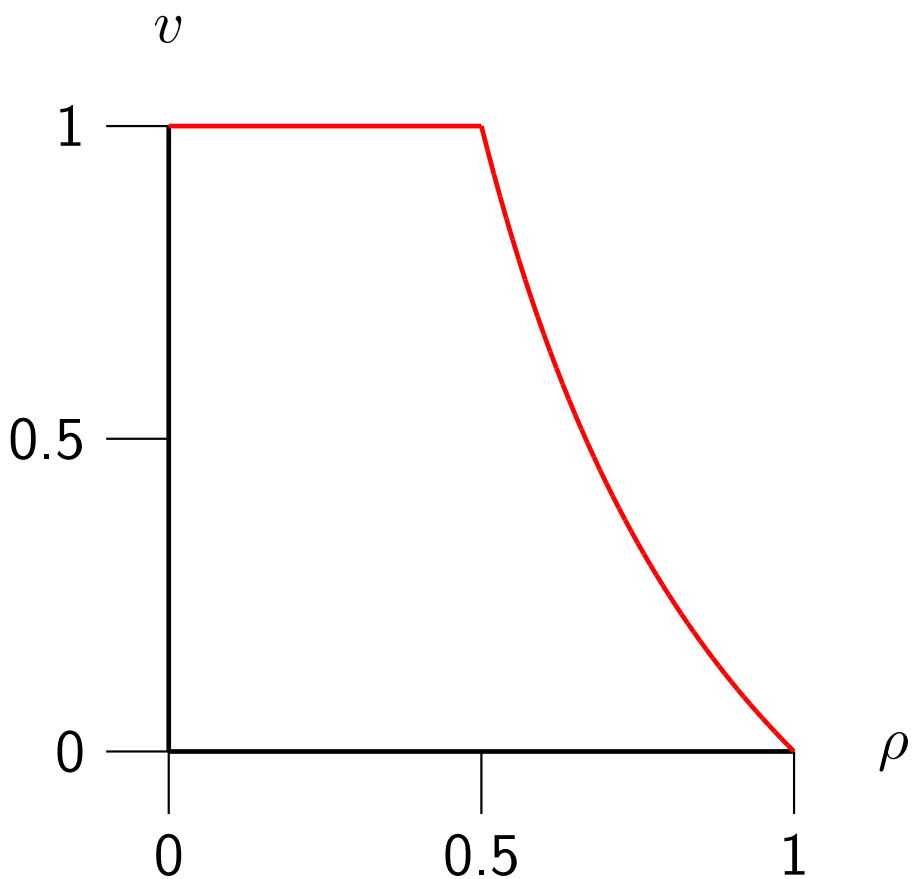
input	111	110	101	100	011	010	001	000
output	1	0	1	1	1	0	0	0

- One car in one cell
- A car can move one cell only if the cell ahead is empty
- All cars move simultaneously

Example of motion

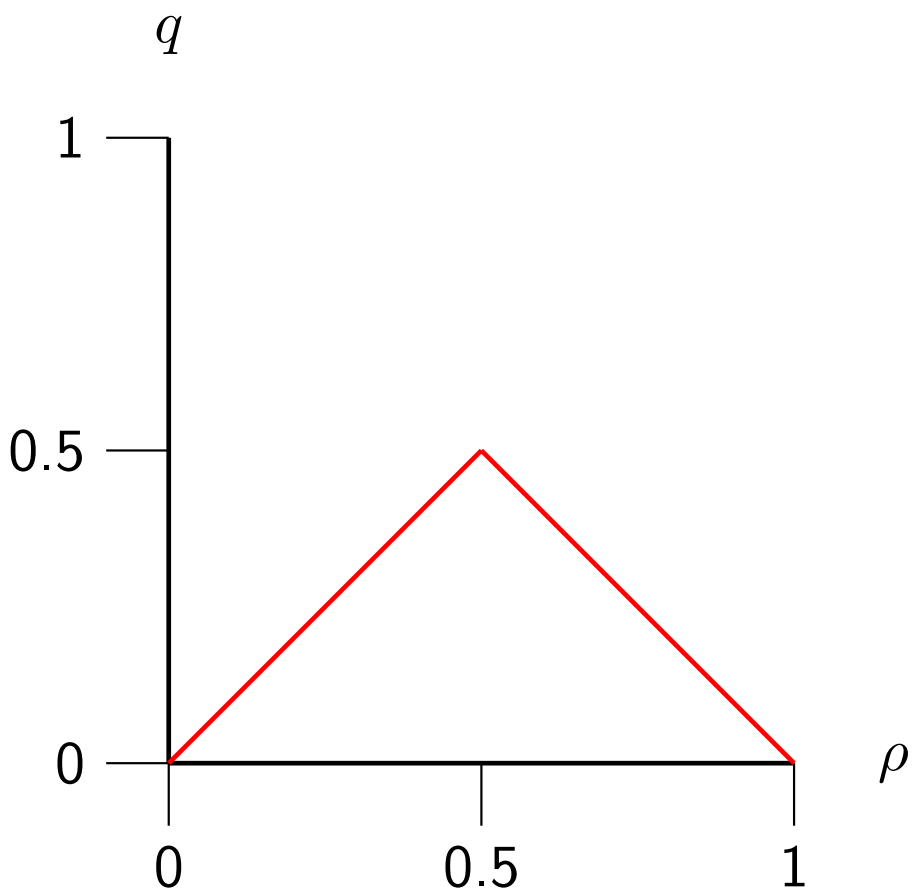


Phase transition at $\rho = 1/2$: Average speed



$$v = \begin{cases} 1 & \rho \leq 1/2 \\ \frac{1}{\rho} - 1 & \text{otherwise} \end{cases} \tag{1}$$

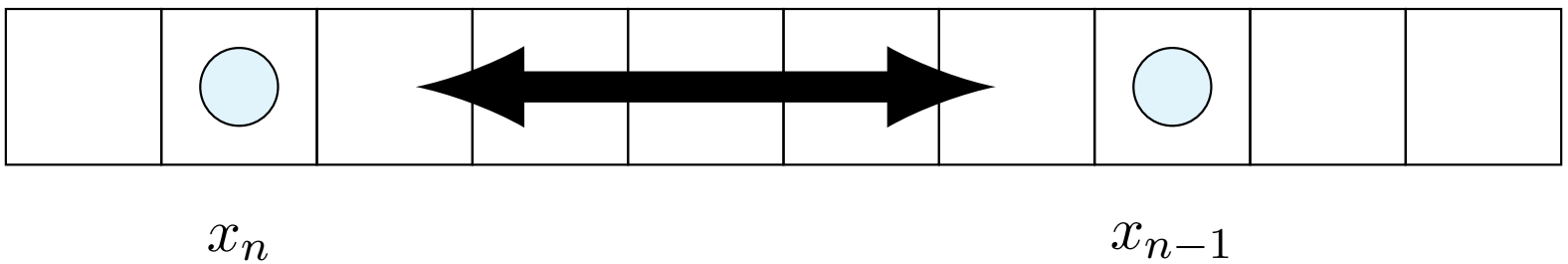
Phase transition at $\rho = 1/2$: Flow



$$q = \begin{cases} \rho & \rho \leq 1/2 \\ 1 - \rho & \text{otherwise} \end{cases} \tag{2}$$

Extended model: Fukui-Ishibashi model

$$g_n = x_{n-1} - x_n - 1$$



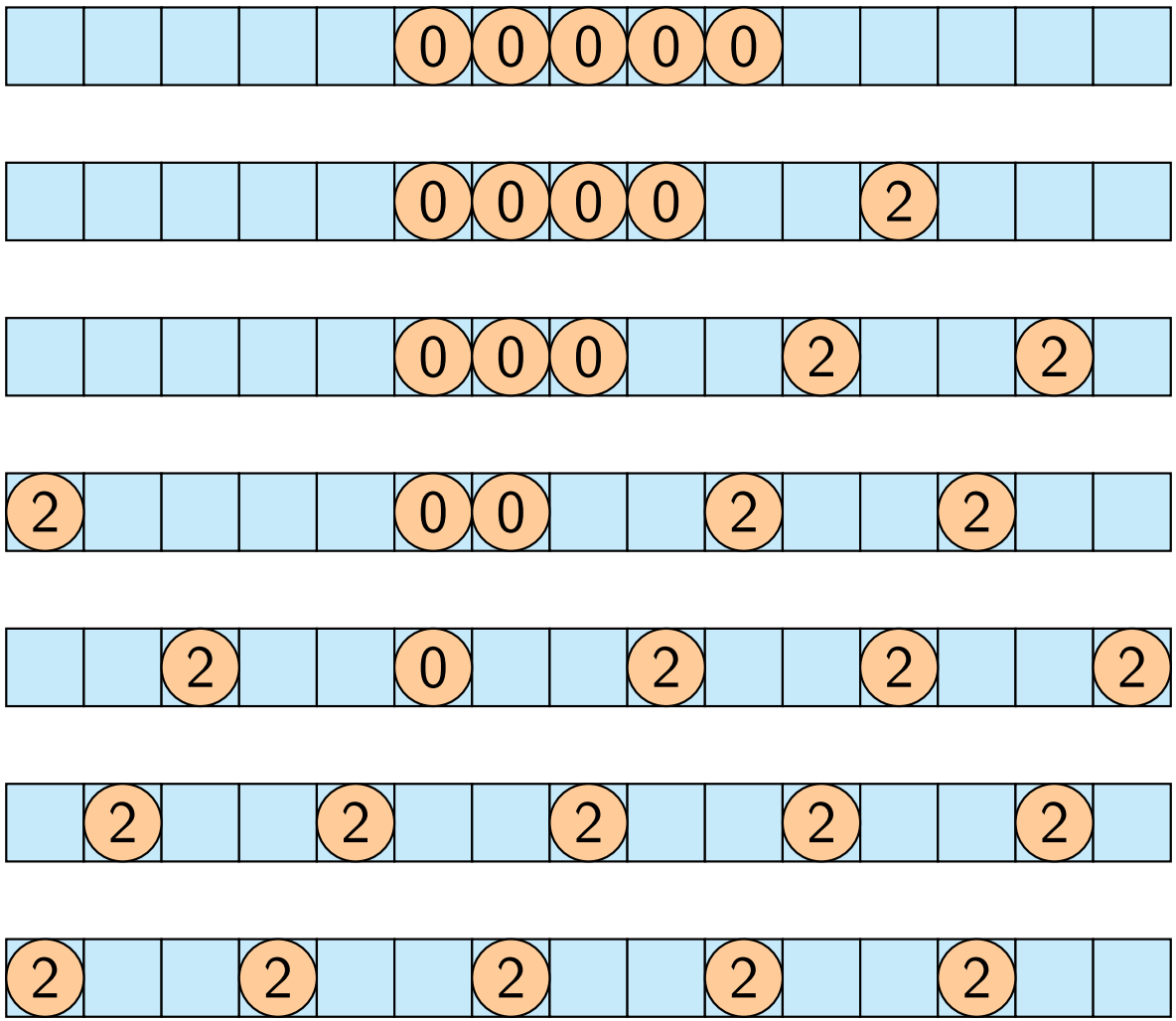
- Maximum speed $v_{\max} \geq 1$
- Move at the maximum speed allowed by the headway distance

$$v'_n = \min\left(g_n, v_{\max}\right) \tag{3}$$

- No acceleration processes!

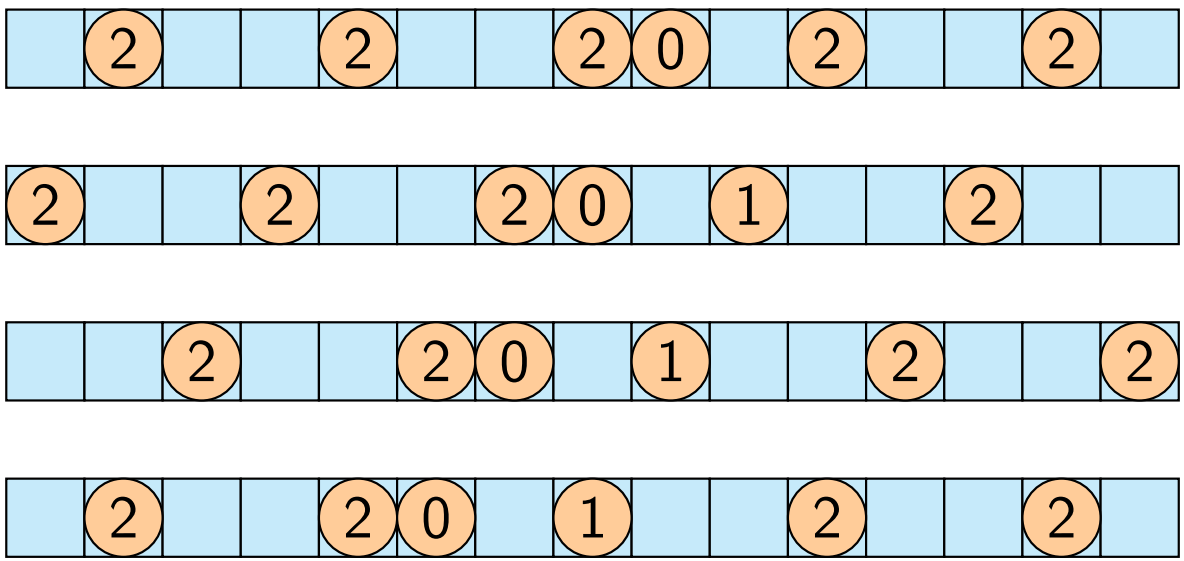
Manual simulation

$$v_{\max} = 2,\ L = 15,\ N = 5$$



Manual simulation

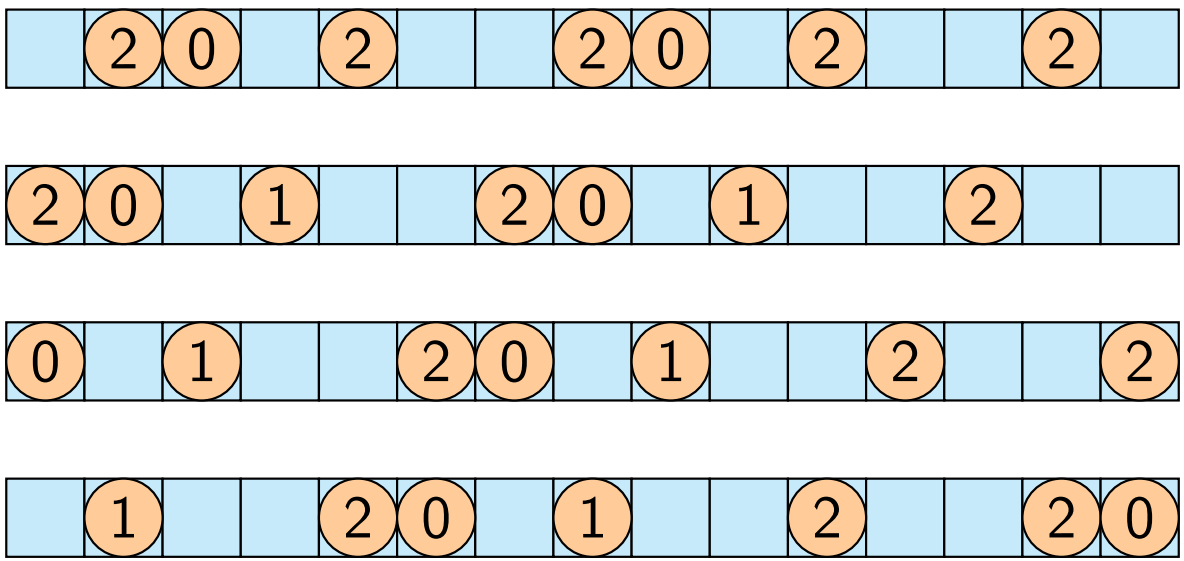
$$v_{\max} = 2,\ L = 15,\ N = 6$$



Sum of speed: $9 = 2 \times 6 - 3 = 2N - 3$

Manual simulation

$$v_{\max} = 2,\ L = 15,\ N = 7$$



Sum of speed: $9 = 2 \times 7 - 6 = 2N - 6$

Theoretical analysis

- $N < 1/3$
  - All cars run at $v = 2$
- $N = L/3 + 1$: Sum of speed: $2N - 3$

$$\rho = \frac{L/3+1}{L} = \frac{1}{3} + \frac{1}{L} \tag{4}$$

$$v = \frac{2N-3}{N} = \frac{2N/L - 3/L}{N/L} = \frac{2\rho - 3\rho + 1}{\rho} = \frac{1-\rho}{\rho} \tag{5}$$

$$q = 1 - \rho \tag{6}$$

- $N = L/3 + 2$: Sum of speed: $2N - 6$

$$\rho = \frac{L/3+2}{L} = \frac{1}{3} + \frac{2}{L} \tag{7}$$

$$v = \frac{2N-6}{N} = \frac{2N/L - 6/L}{N/L} = \frac{2\rho - 3\rho + 1}{\rho} = \frac{1-\rho}{\rho} \tag{8}$$

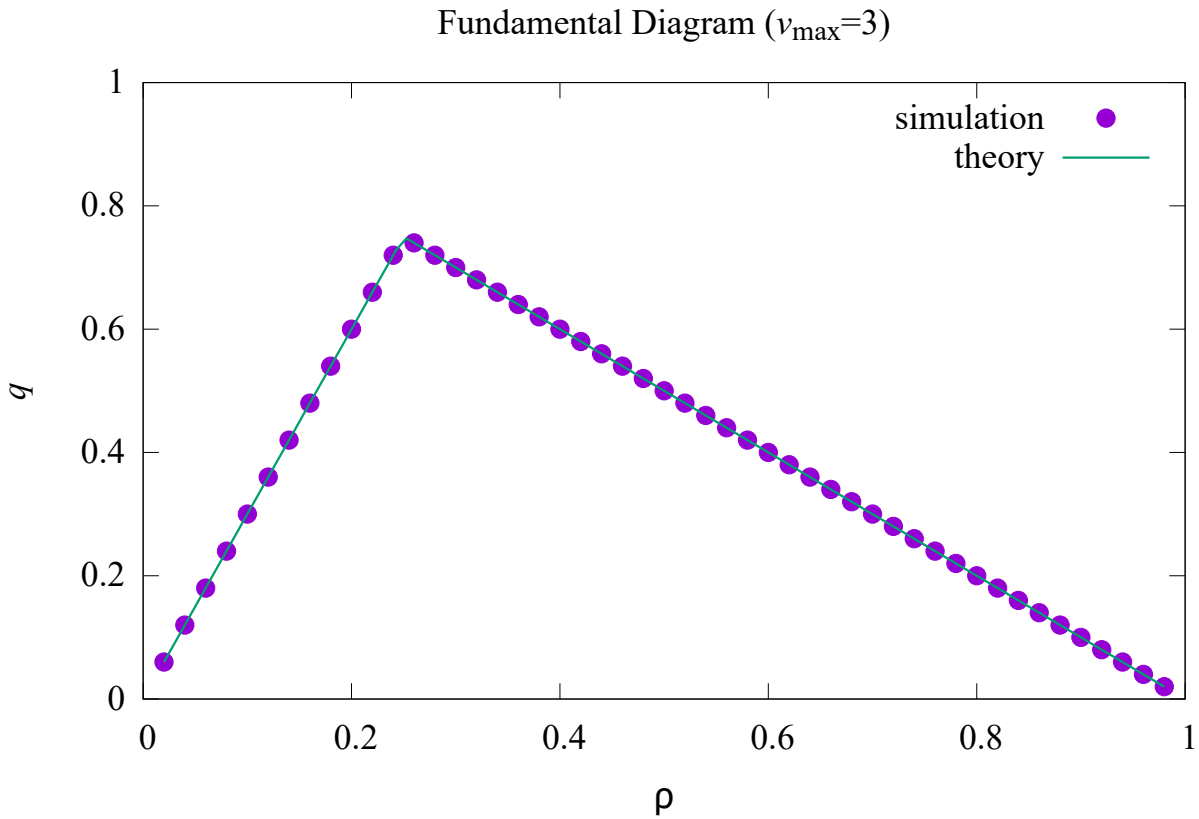
$$q = 1 - \rho \tag{9}$$

Classes

- `Car` class
  - Keep position and speed of a car
  - `evalSpeed()`: decide speed depending on gap
  - `move()`
- `FI` class: Fukui-Ichibashi model
  - `update()`: Calculate speed and move for all cars.
- `Flow` class: Density-Flow relation

`https://github.com/modeling-and-simulation-mc-saga/FukuiIshibashi`

Simulation result



$$q = \begin{cases} v_{\max}\rho & 0 \le \rho < 1/\left(v_{\max}+1\right) \\ 1-\rho & \text{otherwise} \end{cases} \tag{10}$$

Nagel-Schreckenberg model

- Update speed by 3 steps
  - Update simultaneously
  - $\bar{v}$ and $\tilde{v}$ are values in the middle of calculations

1. Accelerate

$$\bar{v} = \min\left(v_n^t + 1, v_{\max}\right) \tag{11}$$

2. Adjust speed by headway

$$\tilde{v} = \min\left(\bar{v}, g_n\right) \tag{12}$$

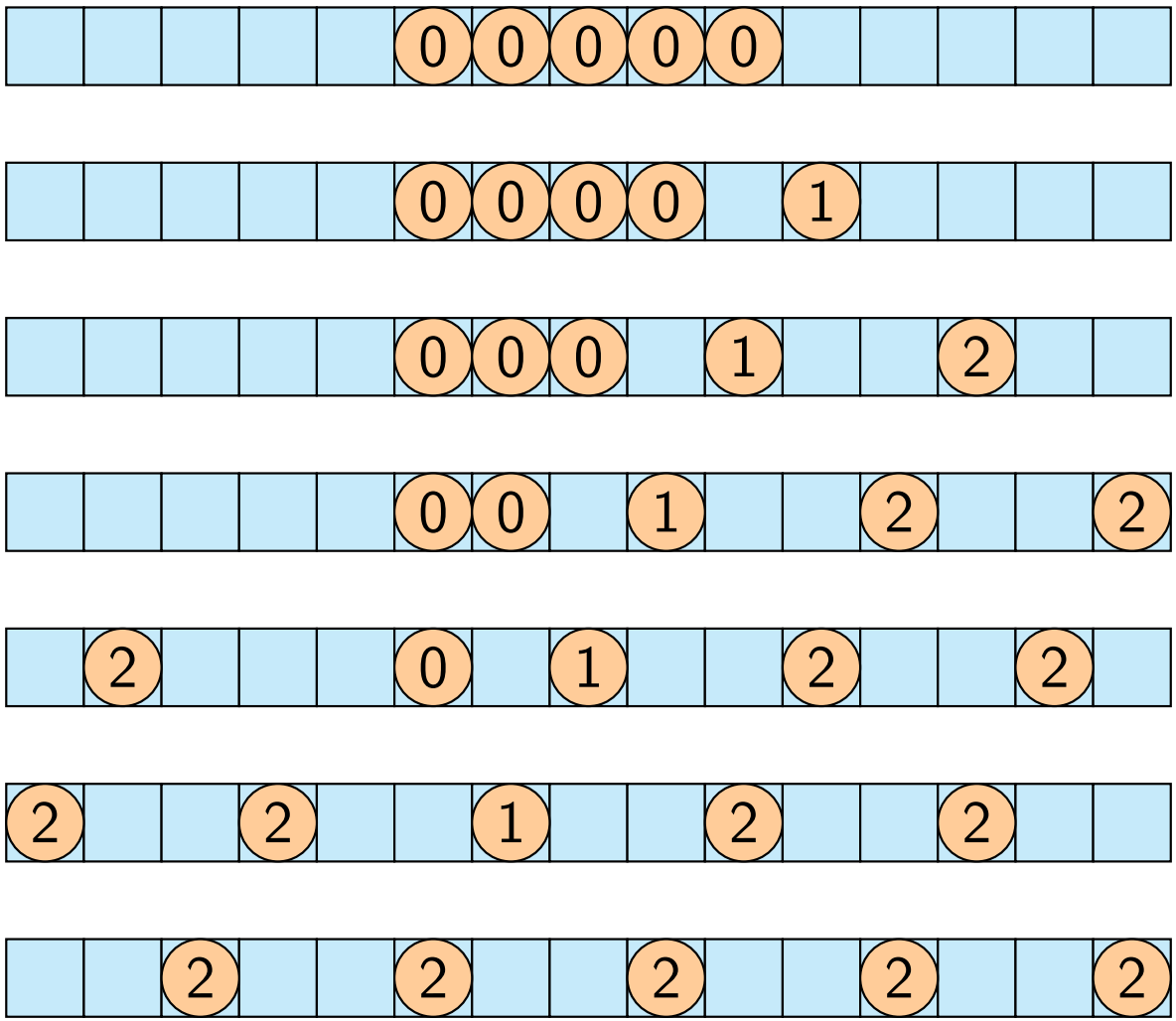
3. Reduce speed by probability p

$$v_n^{t+1} = \max\left(\tilde{v} - 1, 0\right) \tag{13}$$

- Not deterministic

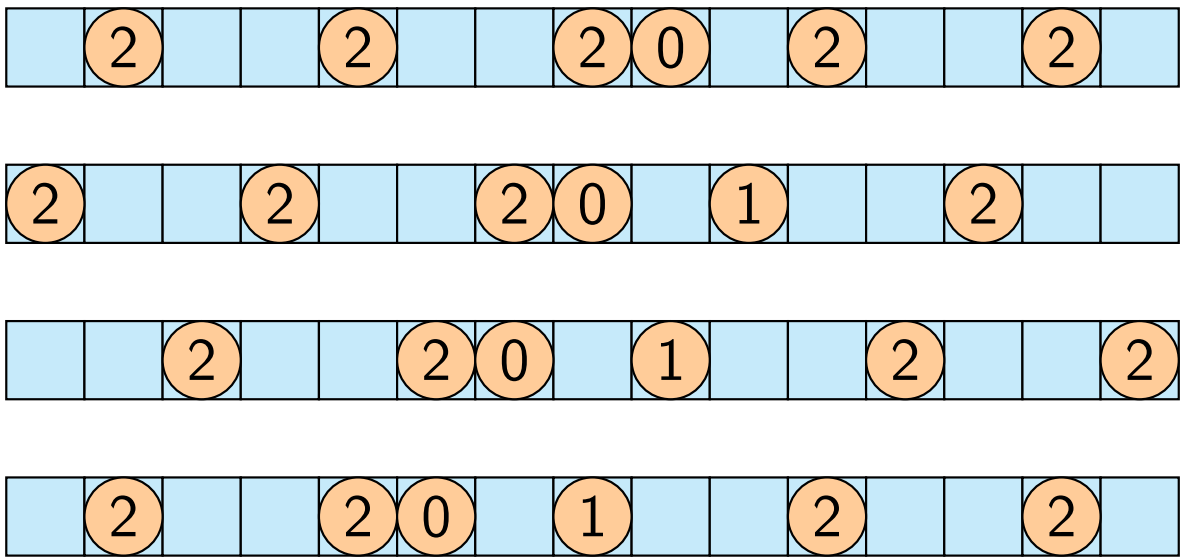
Manual simulation

$$v_{\max} = 2,\ L = 15,\ N = 5,\ p = 0$$



Manual simulation

$$v_{\max} = 2,\ L = 15,\ N = 6,\ p = 0$$



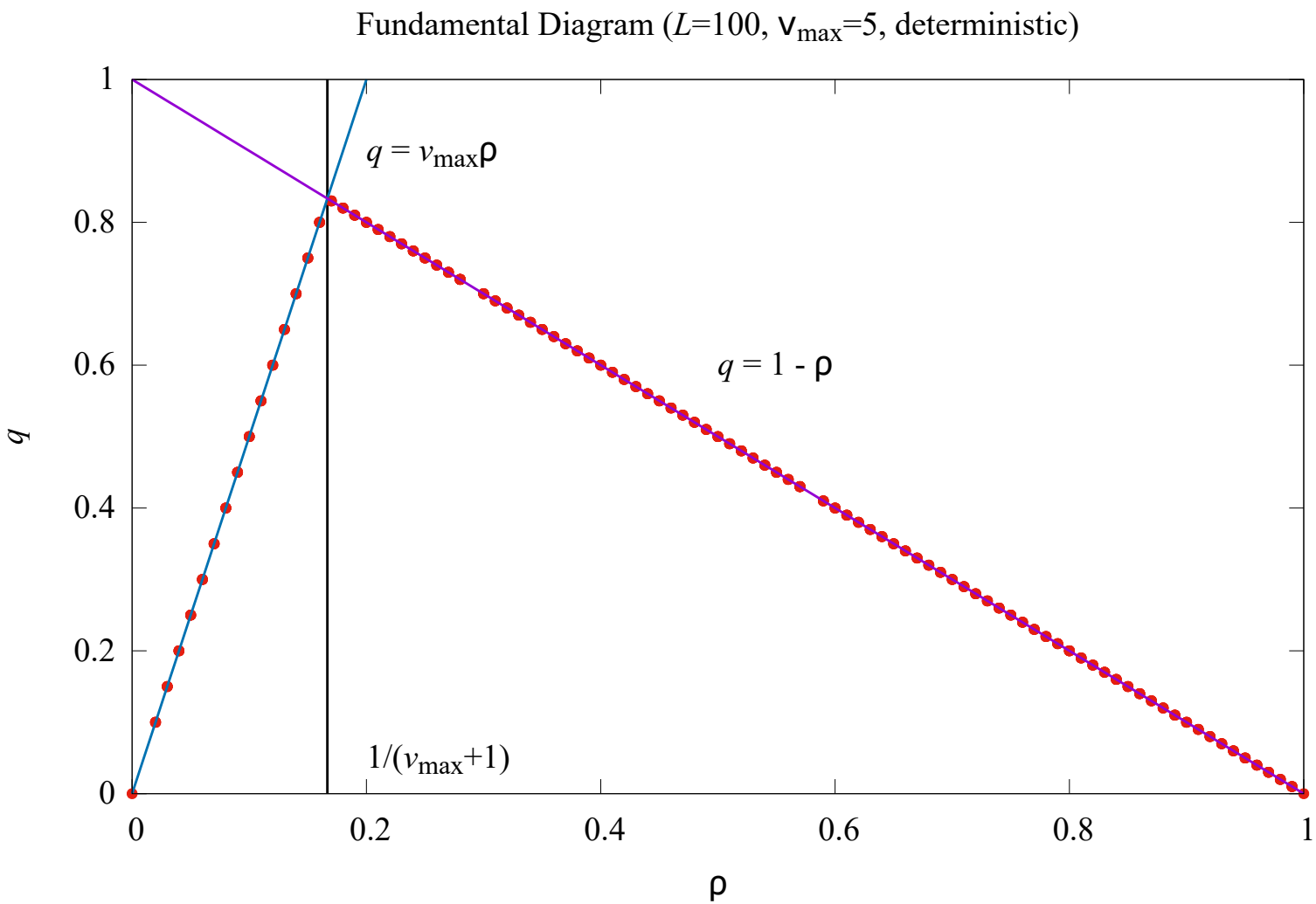
Sum of spped: $9 = 2 \times 6 - 3 = 2N - 3$

Same values of those in Fukui-Ishibashi model

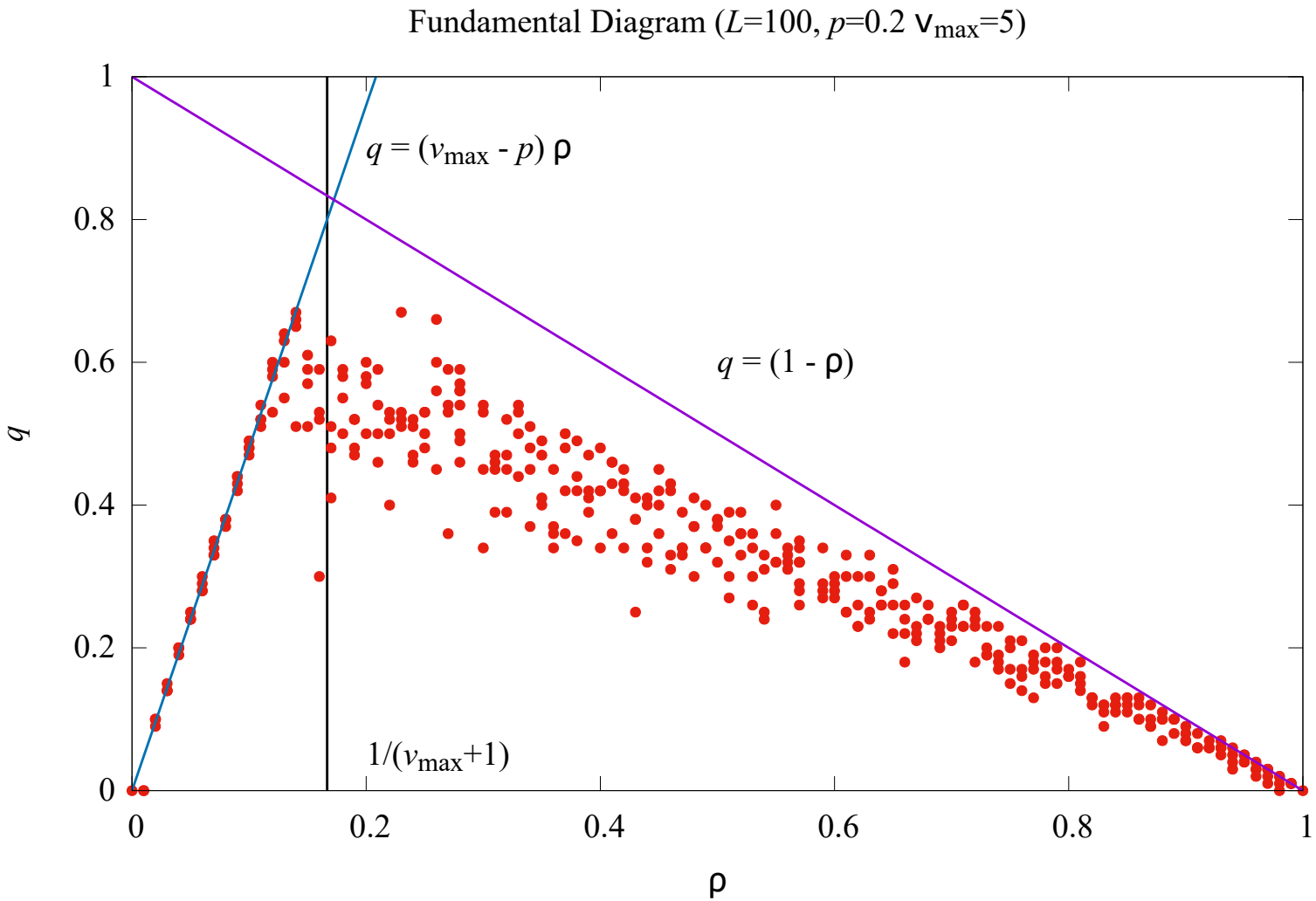
Classes

- `Car` class
- `NaSch` class

`https://github.com/modeling-and-simulation-mc-saga/NaSch`



Same result as Fukui-Ishibashi model


Fundamental Diagram (L=100, p=0.2 v_{max}=5)
$q = (v_{max} - p)\ \rho$
$q = (1 - \rho)$
$1/(v_{max}+1)$
q
ρ
0
0.2
0.4
0.6
0.8
1