Optimal Velocity Traffic Flow Model

モデル化とシミュレーション特論
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Sample Programs

https://github.com/modeling-and-simulation-mc-saga/OV

Car-Following Model

- Car follows the motion of the preceding
  - Keep the same speed of the preceding?
  - Keep the headway to the preceding?
- What should be described?
  - Speed depending on car density
  - Delayed motion

Fundamentals of Optimal Velocity Model

- Optimal speed depending on headway Δx
  - Sigmoidal function of Δx
- Car adjusts its speed by acceleration/deceleration, if its speed deviates from the optimal value.

Optimal Velocity Model

- Position of car: x
- Headway distance to the preceding: Δx

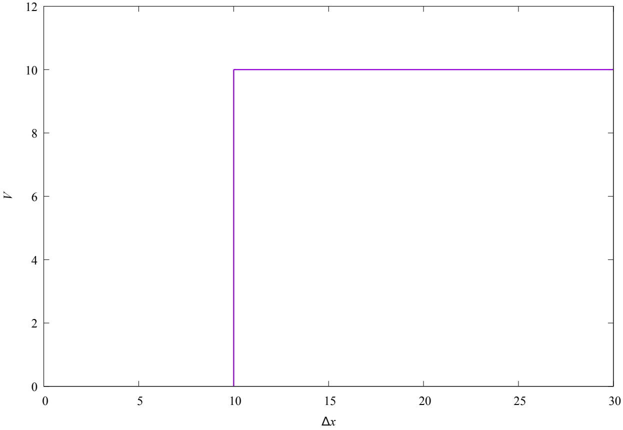
$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = \alpha \left[V_{\mathrm{optimal}} \left(\Delta x \right) - \frac{\mathrm{d}x}{\mathrm{d}t} \right] \tag{1}$$

- Second order differential equation of position
  - Delay in motion naturally introduced

Step OV Function

$$V_{\text{optimal}}(\Delta x) = \begin{cases} v_{\max} & \Delta x > d \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

- N cars on a circuit with length L
  - $b = L/N > d$: All cars run with $v_{\max}$
  - $b < d$: All cars accelerate and decelerate repeatedly


12
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30
Δx

Escape from Jam

- General solution for $V_{\mathrm{optimal}}(\Delta x) = v_{\mathrm{max}}$ (A and B are constants)

$$x(t) = B + v_{\mathrm{max}} t + A e^{-\alpha t} \tag{3}$$

- Verify by deriving the first and second derivative

$$\frac{\mathrm{d}x}{\mathrm{d}t} = v_{\mathrm{max}} - \alpha A e^{-\alpha t} \tag{4}$$

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = \alpha^2 A e^{-\alpha t} \tag{5}$$

- Two cars stopping at a distance Δx_{J}
- The leading car starts at $t = 0$ because $\Delta x > d$

$$x^{\mathrm{P}}(t) = \Delta x_{\mathrm{J}} + v_{\max} t - \frac{v_{\max}}{\alpha}\left(1 - e^{-\alpha t}\right) \tag{6}$$

$$x^{\mathrm{P}}(0) = \Delta x_{\mathrm{J}} \tag{7}$$

$$v^{\mathrm{P}}(t) = v_{\max}\left(1 - e^{-\alpha t}\right) \tag{8}$$

$$v^{\mathrm{P}}(0) = 0 \tag{9}$$

- The follower car starts at $t = t_0$ because $\Delta x > d$

$$\Delta x_{\mathrm{J}} + v_{\max} t_0 - \frac{v_{\max}}{\alpha}\left(1 - e^{-\alpha t_0}\right) = d \tag{10}$$

- Trajectory of the follower

$$x^{\mathrm{F}}(t) = v_{\max}(t - t_0) - \frac{v_{\max}}{\alpha}\left(1 - e^{-\alpha(t-t_0)}\right) \tag{11}$$

$$x^{\mathrm{F}}(t_0) = 0 \tag{12}$$

$$v^{\mathrm{F}}(t) = v_{\max}\left(1 - e^{-\alpha(t-t_0)}\right) \tag{13}$$

$$v^{\mathrm{F}}(t_0) = 0 \tag{14}$$

- Headway of the follower

$$\begin{aligned}\Delta x(t) &= x^{\mathrm{P}}(t) - x^{\mathrm{F}}(t) \\ &= \Delta x_{\mathrm{J}} + v_{\max} t_0 + \frac{v_{\max}}{\alpha} e^{-\alpha t}\left(1 - e^{\alpha t_0}\right) \\ &\xrightarrow[t\to\infty]{} \Delta x_{\mathrm{J}} + v_{\max} t_0\end{aligned} \tag{15}$$

Catch up to Jam

- General solution for $V_{\mathrm{optimal}}(\Delta x) = 0$ (A and B are constants)

$$x(t) = B + Ae^{-\alpha t} \tag{16}$$

- Verify by deriving the first and second derivative

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\alpha A e^{-\alpha t} \tag{17}$$

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = \alpha^2 A e^{-\alpha t} \tag{18}$$

- Two car running at a distance Δx_{F}
- The leader car starts to decelerate at $t = 0$ because $\Delta x < d$
- Trajectory of the leader

$$x^{\mathrm{P}}(t) = \Delta x_{\mathrm{F}} + \frac{v_{\max}}{\alpha}\left(1 - e^{-\alpha t}\right) \tag{19}$$

$$x^{\mathrm{P}}(0) = \Delta x_{\mathrm{F}} \tag{20}$$

$$v^{\mathrm{P}}(t) = v_{\max} e^{-\alpha t} \tag{21}$$

$$v^{\mathrm{P}}(0) = v_{\max} \tag{22}$$

- The follower starts to decelerate at $t = t'$ because $\Delta x < d$
- Trajectory of the follower

$$x^{\mathrm{F}}(t) = v_{\max} t' + \frac{v_{\max}}{\alpha}\left(1 - e^{-\alpha(t-t')}\right) \tag{23}$$

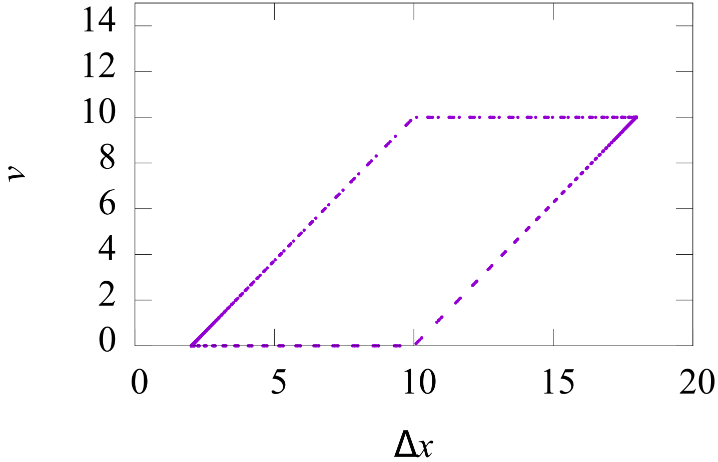
$$x^{\mathrm{F}}(t') = v_{\max} t' \tag{24}$$

$$v^{\mathrm{F}}(t) = v_{\max} e^{-\alpha(t-t')} \tag{25}$$

$$v^{\mathrm{F}}(t') = v_{\max} \tag{26}$$

- Headway of the follower

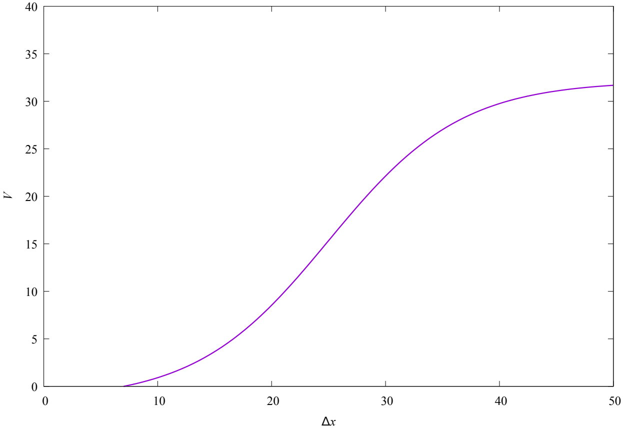
$$\begin{aligned}
\Delta x &= x^{\mathrm{P}}(t) - x^{\mathrm{F}}(t) \\
&= \Delta x_{\mathrm{F}} + \frac{v_{\max}}{\alpha}\left(1 - e^{-\alpha t}\right) - v_{\max} t' + \frac{v_{\max}}{\alpha}\left(1 - e^{-\alpha(t-t')}\right) \\
&= \Delta x_{\mathrm{F}} - v_{\max} t' + \frac{v_{\max}}{\alpha} e^{-\alpha t}\left(1 - e^{\alpha t'}\right) \\
&\underset{t\to\infty}{\longrightarrow} \Delta x_{\mathrm{F}} - v_{\max} t'
\end{aligned} \tag{27}$$


14
12
10
8
6
4
2
0
0
5
10
15
20
v
Δx

Realistic OV Function

$$V_{\mathrm{optimal}}\left(\Delta x\right) = \frac{v_{\max}}{2}\left[\tanh\left(2\frac{\Delta x - d}{w}\right) + c\right] \tag{28}$$

parameters	values
$v_{\max}$	33.6 m/s
d	25 m
w	23.3 m
c	0.913
α	2 1/s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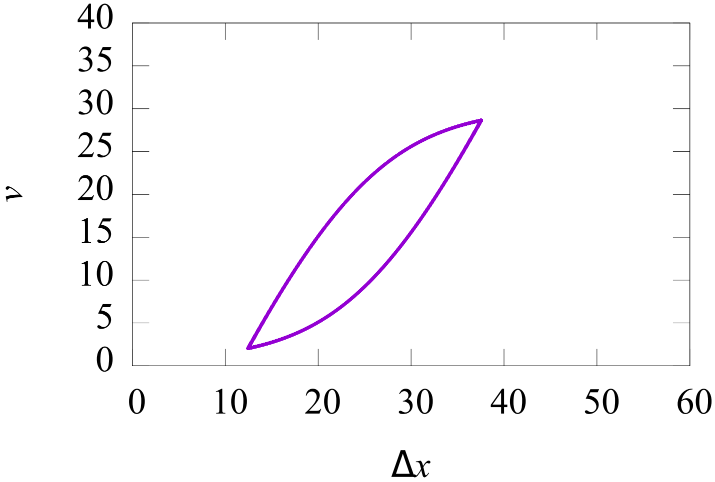
30
40
50
Δx


40
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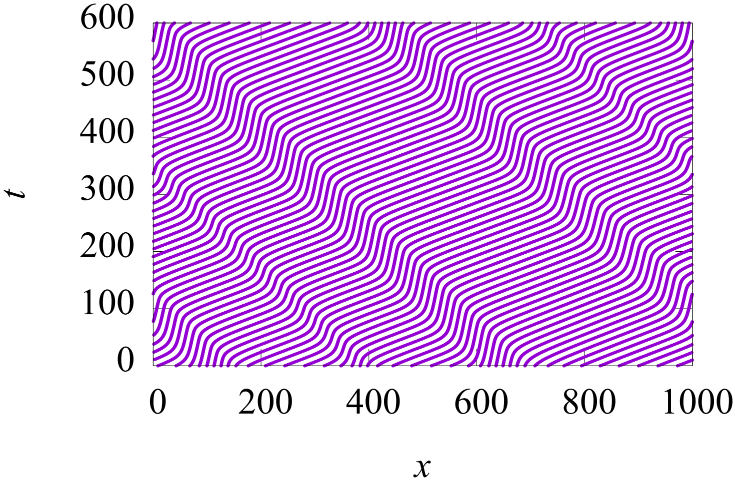
40
50
60
Δx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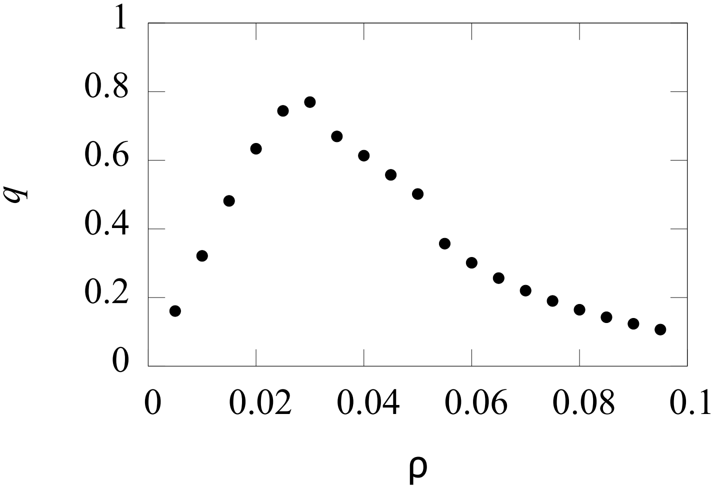
1
0.8
0.6
0.4
0.2
0
q
0
0.02
0.04
0.06
0.08
0.1
ρ

Class plan

- abstractModel package
    - Car
        - Keep position and speed at fixed time interval
        - Not describe motion
    - OV
        - Move car by given OV function
- analysis package
    - Fundamental
        - Generate fundamental diagrams
    - HV
        - Generate trajectory in headway-speed plane

- models package
  - Simulation
    - Execute simulation with given OV function
    - OV function is given as `DoubleFunction<Double>`.
  - Step
    - Simulation with step OV function
  - Tanh
    - Simulation with tanh OV function

Example: Step OV function

```
public static void main(String args[]) throws IOException {
    int length = 1000;
    int tmax = 10000;
    double alpha = 1.;
    double vmax = 10.;
    double d = 10.;
    int numCar = 100;
    DoubleFunction<Double> ovfunction
            = x -> {
                double v = 0.;
                if (x > d) { v = vmax; }
                return v;
            };
    Simulation sys
        = new Simulation(ovfunction, length, numCar, alpha);
    sys.spacetime("Step-spacetime.txt");
    sys.hv("Step-hv.txt");
    sys.fundamental("Step-fundamental.txt", 10, 190, 10, 100);
}
```