Chaos and Logistic Map : part2

モデル化とシミュレーション特論
2021 年度前期
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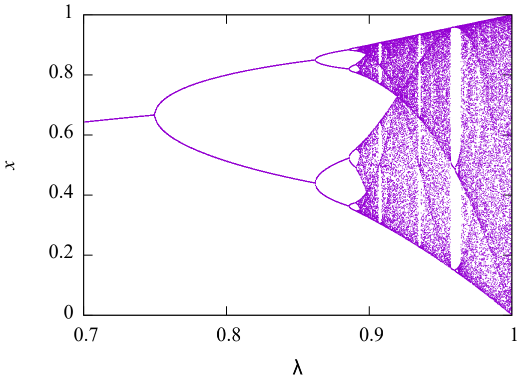
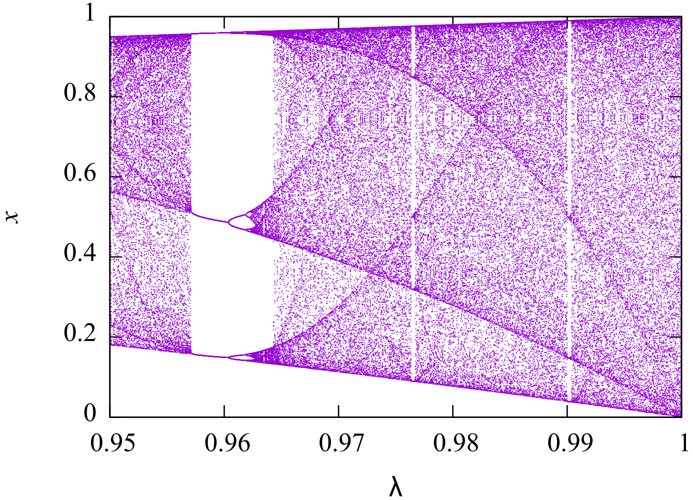
Period doubling to chaos

- Trajectories are doubled repeatedly by increasing λ
- Period becomes infinite at $\lambda \simeq 0.893$



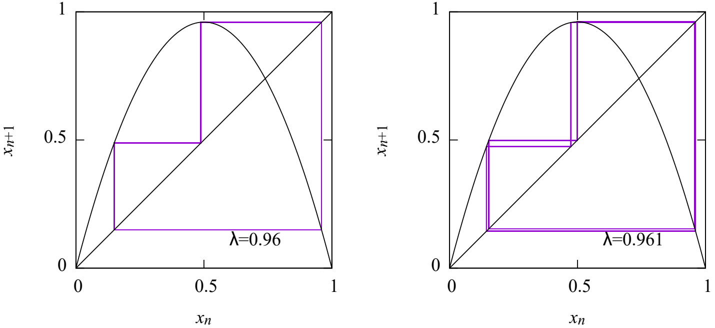
- For $\lambda > 0.893$, trajectories show band structure.
  - Not periodic, not random
  - Non-uniform density of trajectories

Period-3 region



You can also see period-5 and 7 windows.

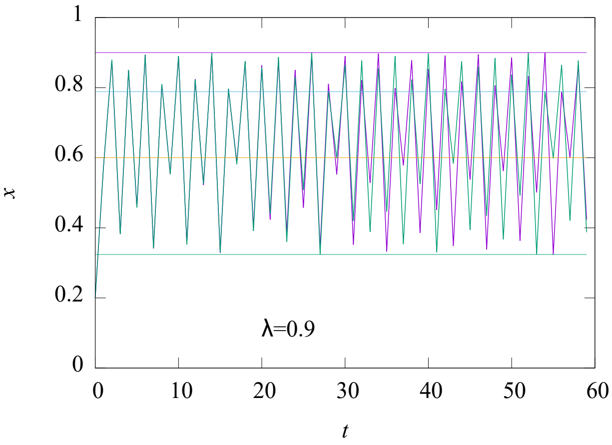
Period-3 orbit



- Period-3 trajectories near $\lambda \sim 0.96$
- Period doubled to period-6 trajectories

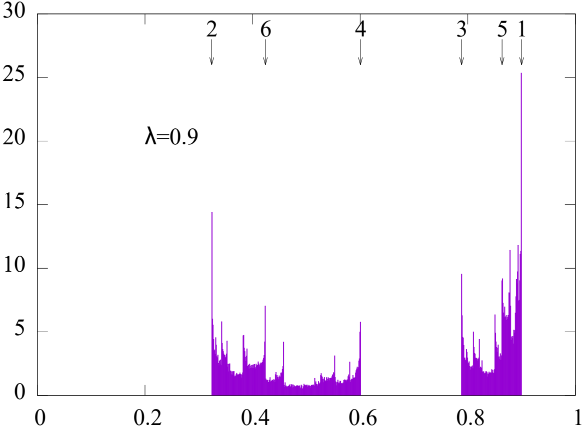
Chaotic motions

- small difference in initial values expands
- finally two trajectories seem to behave independently



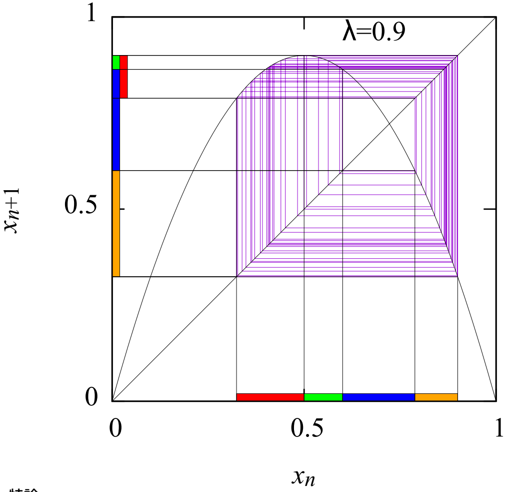
Non-uniform density of trajectories

$$f_\lambda^{[n]}\left(\frac{1}{2}\right)$$



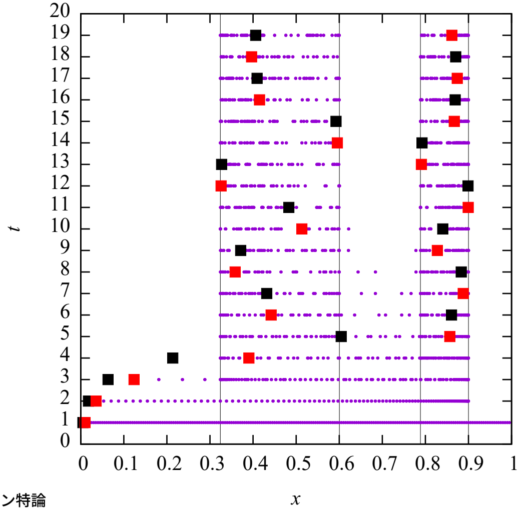
Bands of trajecctories

- Bands of trajectories are expended and folded.
- This is the origin of chaotic motion.



Uniform initial points are absorbed into two bands

- Two points ■ and ■, which are initially close each other, separate and behave almost independently.



Super-stable point: $x = 1/2$

$$f_\lambda(x) = 4\lambda(1 - x)$$

$$f'_\lambda(x) = 4\lambda(1 - 2x)$$

$$\frac{\mathrm{d}}{\mathrm{d}x} f_\lambda^{[2]}(x) = f'_\lambda(f_\lambda(x)) \cdot \frac{\mathrm{d}}{\mathrm{d}x} f_\lambda(x) \quad (1)$$

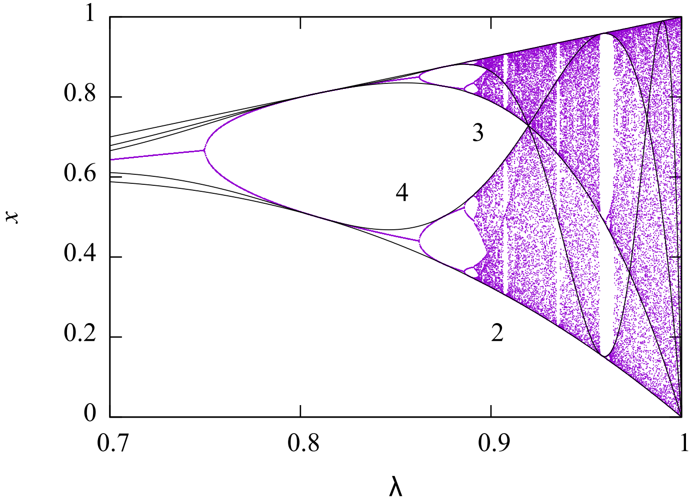
$$\frac{\mathrm{d}}{\mathrm{d}x} f_\lambda^{[n]}(x) = f'_\lambda\left(f_\lambda^{[n-1]}(x)\right) \cdot \frac{\mathrm{d}}{\mathrm{d}x} f_\lambda^{[n-1]}(x) \quad (2)$$

$$f'_\lambda\left(\frac{1}{2}\right) = 4\lambda\left(1 - 2\frac{1}{2}\right) = 0 \tag{3}$$

$$\begin{aligned}\left.\frac{\mathrm{d}}{\mathrm{d}x}f_\lambda^{[2]}(x)\right|_{x=x_0=1/2} &= f'_\lambda\left(f_\lambda(x)\right)\cdot\left.\frac{\mathrm{d}}{\mathrm{d}x}f_\lambda(x)\right|_{x=x_0=1/2} \\ &= f'_\lambda(x_1)\cdot f'_\lambda(x_0) = 0 \end{aligned} \tag{4}$$

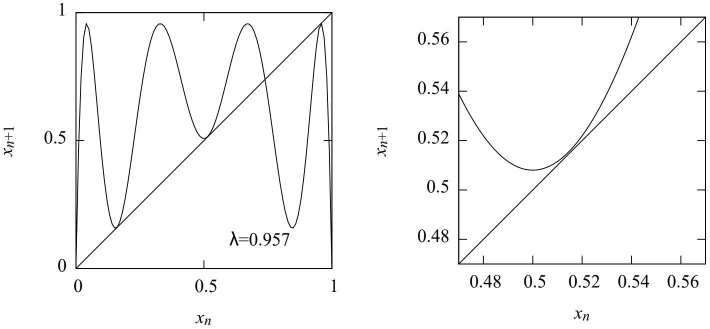
$$\begin{aligned}\left.\frac{\mathrm{d}}{\mathrm{d}x}f_\lambda^{[n]}(x)\right|_{x=x_0=1/2} &= f'_\lambda\left(f_\lambda(x_{n-1})\right)\cdot\left.\frac{\mathrm{d}}{\mathrm{d}x}f_\lambda^{[n-1]}(x)\right|_{x=x_0=1/2} \\ &= \prod_{k=0}^{n-1} f'_\lambda(x_k) = 0 \end{aligned} \tag{5}$$

Trajectories of $x = 1/2$ are keys to understand band structure

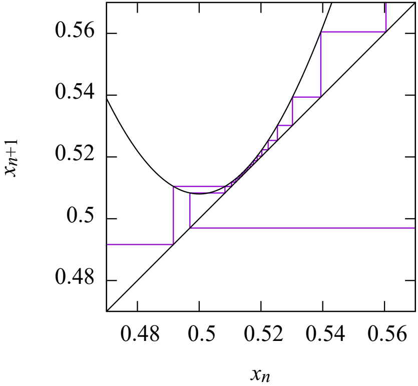


Tangent Bifurcation

- $\lambda_{\rm C}$: period-3 trajectories emerges
- A little bit lower λ than $\lambda_{\rm C}$
- $f_{\lambda}^{[3]}(x)$ does not intersect with $y = x$ line. There are narrow corridor.

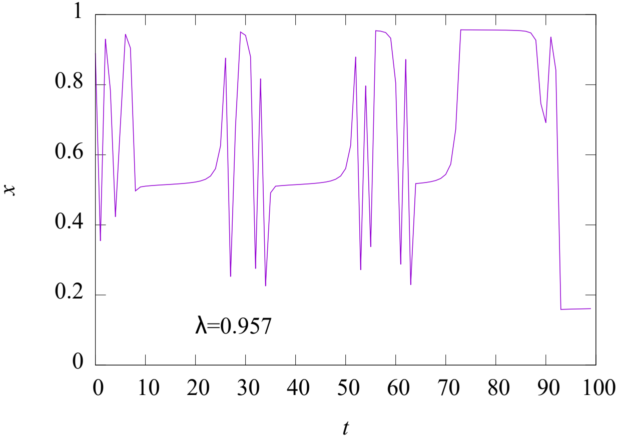


- Trajectories (per 3 times) stays long time at the narrow corridor



Intermittency

- After staying in the narrow corridor, trajectories varies widely.



Note: x values are plotted every 3 steps.