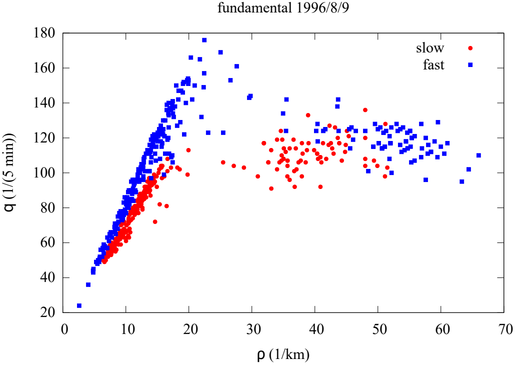
Cellular Automaton Traffic Flow Model

モデル化とシミュレーション特論
2023 年度前期

佐賀大学理工学研究科　只木進一

Fundamental Diagram: density-flow relations

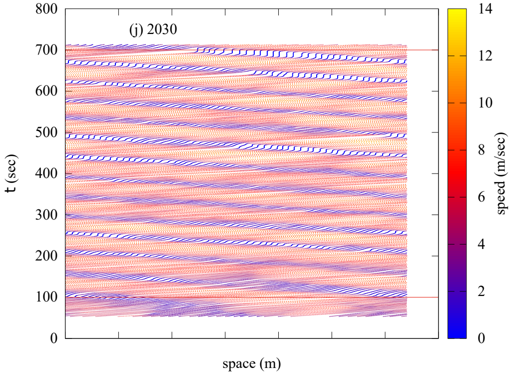
Data observed at a point on Tomei Expressway



- Unongested and congested traffic
- Broad distribution of data points for congested flows

Spacetime Diagram

Data from experiments at Nagoya Dome Stadium



- Cars run rightward.
- Jam clusters propagate backward.

Discrete model of traffic flow

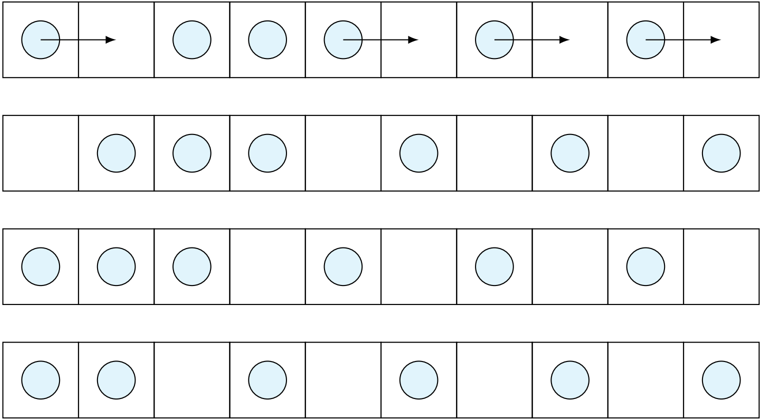
- Cars have finite length (around 3 to 4m for passenger cars)
- No two cars can occupy the same space simultaneously (volume exclusion effect, **排除体積効果**)
- A car follows the motion of the preceding car with a certain delay

Rule 184: Simplest model

input	111	110	101	100	011	010	001	000
output	1	0	1	1	1	0	0	0

- One cell can contain a maximum of one car (0 or 1)
- A car can only move to the next cell if it is empty
- All cars move simultaneously

Example of motion



Oservation

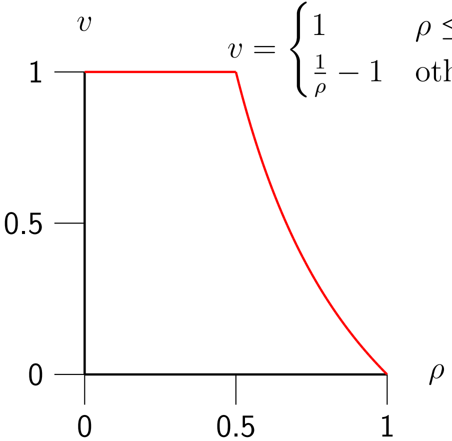
- Average speed
  - Changes in cell values: $10 \to 01$ corresponds to a motion of one car
  - N_{m}: the number of cells that change values

$$v = \frac{N_{\mathsf{m}}/2}{N} \tag{2.1}$$

- Flow
  - $q = \rho v$
  - $\rho = N/L$

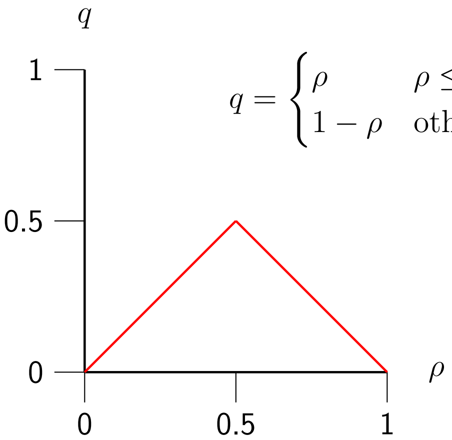
$$q = \frac{N_{\mathsf{m}}/2}{L} \tag{2.2}$$

Phase transition at $\rho = 1/2$: Average speed



$$v = \begin{cases} 1 & \rho \leq 1/2 \\ \frac{1}{\rho} - 1 & \text{otherwise} \end{cases} \tag{2.3}$$

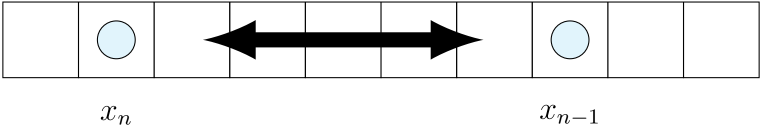
Phase transition at $\rho = 1/2$: Flow



$$q = \begin{cases} \rho & \rho \leq 1/2 \\ 1 - \rho & \text{otherwise} \end{cases} \tag{2.4}$$

Extended model: Fukui-Ishibashi model

$$g_n = x_{n-1} - x_n - 1$$



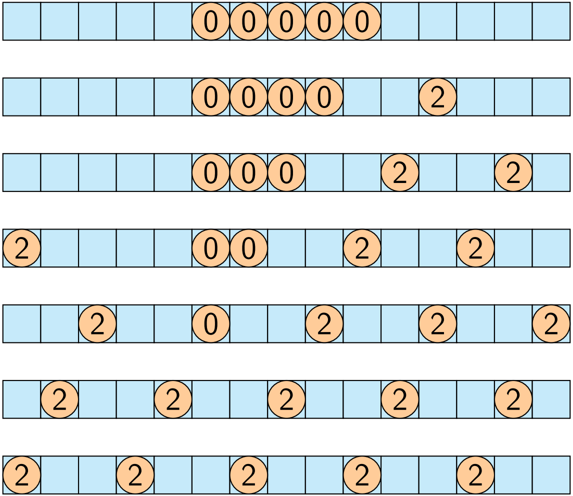
- The maximum speed: $v_{\max} \geq 1$
- Move at the maximum speed allowed by the headway distance

$$v'_n = \min\left(g_n, v_{\max}\right) \qquad (3.1)$$

- No acceleration processes involved!

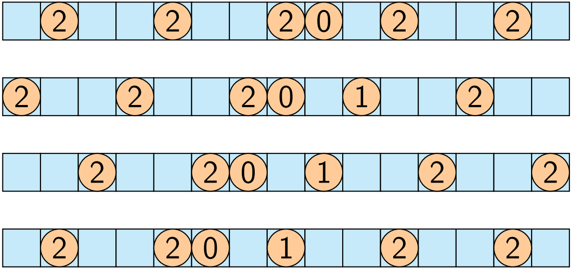
Manual simulation

$$v_{\max} = 2,\ L = 15,\ N = 5$$



Manual simulation

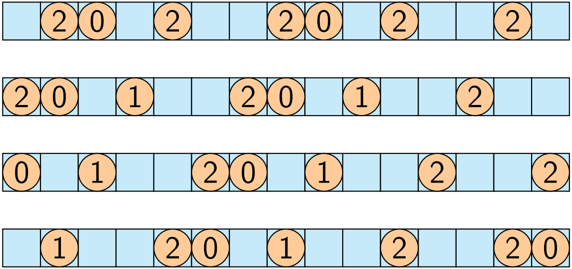
$v_{\max} = 2, \ L = 15, \ N = 6$



Sum of speed: $9 = 2 \times 6 - 3 = 2N - 3$

Manual simulation

$v_{\max} = 2,\ L = 15,\ N = 7$



Sum of speed: ~~9~~ 8 $= 2 \times 7 - 6 = 2N - 6$

Theoretical analysis

- $N < 1/3$
  - All cars run at $v = 2$
- $N = L/3 + 1$: Sum of speed: $2N - 3$

$$\rho = \frac{L/3+1}{L} = \frac{1}{3} + \frac{1}{L} \quad (3.2)$$

$$v = \frac{2N-3}{N} = \frac{2N/L - 3/L}{N/L} = \frac{2\rho - 3\rho + 1}{\rho} = \frac{1-\rho}{\rho} \quad (3.3)$$

$$q = 1 - \rho \quad (3.4)$$

- $N = L/3 + 2$: Sum of speed: $2N - 6$

$$\rho = \frac{L/3 + 2}{L} = \frac{1}{3} + \frac{2}{L} \tag{3.5}$$

$$v = \frac{2N - 6}{N} = \frac{2N/L - 6/L}{N/L} = \frac{2\rho - 3\rho + 1}{\rho} = \frac{1 - \rho}{\rho} \tag{3.6}$$

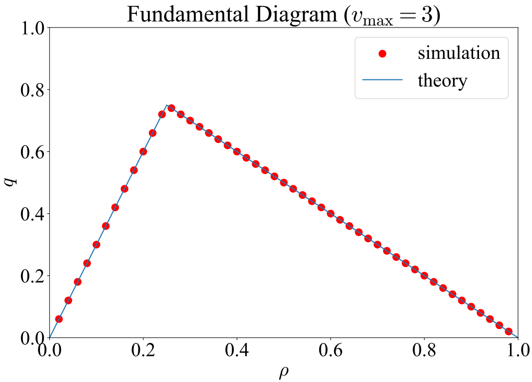
$$q = 1 - \rho \tag{3.7}$$

Classes

- Car class
  - Keep position and speed of a car
  - `evalSpeed()`: decide speed depending on gap
  - `move()`
- `FI` class: Fukui-Ichibashi model
  - `update()`: Calculate speed and move for all cars.
- `Flow` class: Density-Flow relation

`https://github.com/modeling-and-simulation-mc-saga/FukuiIshibashi`

Simulation result



$$q = \begin{cases} v_{\max}\rho & 0 \leq \rho < 1/\left(v_{\max}+1\right) \\ 1-\rho & \text{otherwise} \end{cases} \tag{4.1}$$

Nagel-Schreckenberg model

- Update speed by 3 steps
  - Update simultaneously
  - $\bar{v}$ and $\tilde{v}$ are values in the middle of calculations

1. Accelerate

$$\bar{v} = \min\left(v_n^t + 1, v_{\max}\right) \tag{5.1}$$

2. Adjust speed by headway

$$\tilde{v} = \min\left(\bar{v}, g_n\right) \tag{5.2}$$

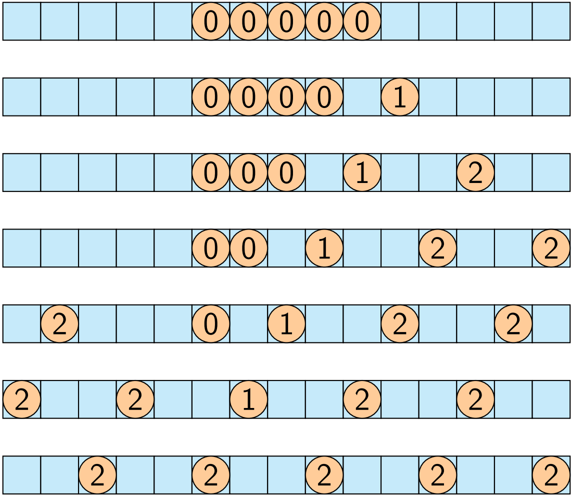
3. Reduce speed by probability p

$$v_n^{t+1} = \max\left(\tilde{v} - 1, 0\right) \tag{5.3}$$

- Not deterministic

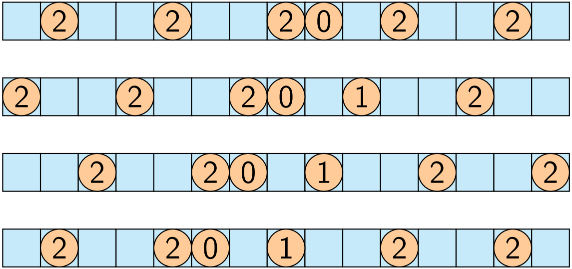
Manual simulation

$$v_{\max} = 2,\ L = 15,\ N = 5,\ p = 0$$



Manual simulation

$$v_{\max} = 2,\ L = 15,\ N = 6,\ p = 0$$



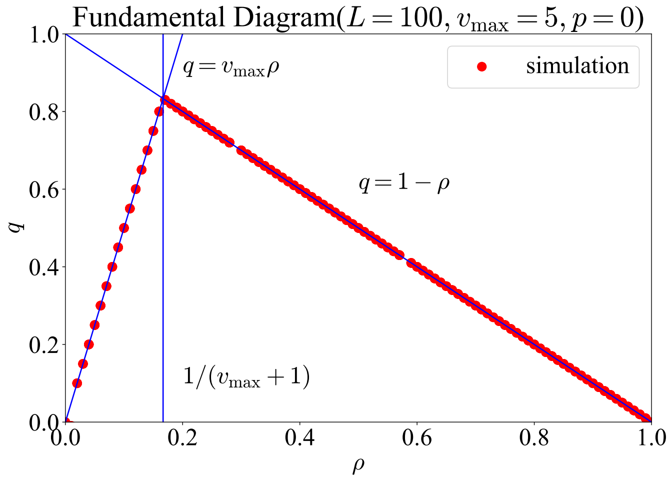
Sum of speed: $9 = 2 \times 6 - 3 = 2N - 3$

Same values of those in Fukui-Ishibashi model

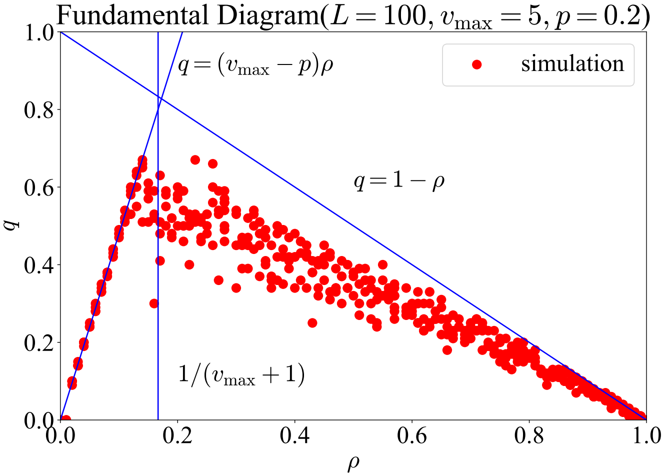
Classes

- Car class
- NaSch class

https://github.com/modeling-and-simulation-mc-saga/NaSch



Same result as Fukui-Ishibashi model


Fundamental Diagram$(L=100, v_{\max}=5, p=0.2)$
1.0
0.8
0.6
0.4
0.2
0.0
q
0.0
0.2
0.4
0.6
0.8
1.0
ρ
simulation
$q=(v_{\max}-p)\rho$
$q=1-\rho$
$1/(v_{\max}+1)$