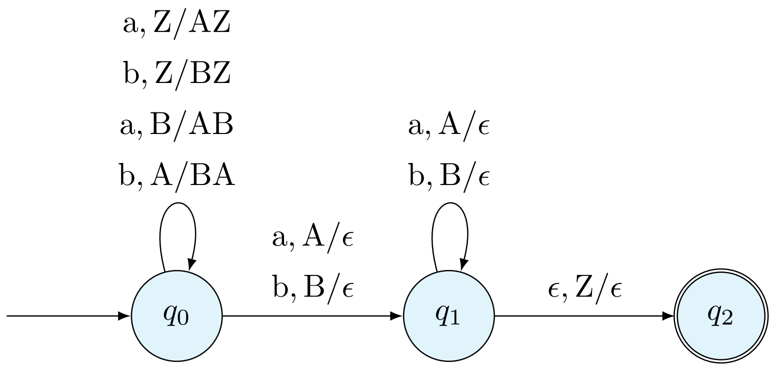
「離散数学・オートマトン」演習問題 13 （解答例)

2020/1/19

1　プッシュダウンオートマトン

課題 1　以下のような決定性有限オートマトン M を考える。



$$
\begin{aligned}
Q &= \{q_0.q_1, q_2\} \\
\Sigma &= \{\mathrm{a}, \mathrm{b}\} \\
\Gamma &= \{\mathrm{A}, \mathrm{B}, \mathrm{Z}\} \\
F &= \{q_2\}
\end{aligned}
$$

$$
\begin{aligned}
&\delta\left(q_0, \mathrm{a}, \mathrm{Z}\right) = \left(q_0, \mathrm{AZ}\right), && \delta\left(q_0, \mathrm{b}, \mathrm{Z}\right) = \left(q_0, \mathrm{BZ}\right), \\
&\delta\left(q_0, \mathrm{a}, \mathrm{B}\right) = \left(q_0, \mathrm{AB}\right), && \delta\left(q_0, \mathrm{b}, \mathrm{A}\right) = \left(q_0, \mathrm{BA}\right), \\
&\delta\left(q_0, \mathrm{a}, \mathrm{A}\right) = \left(q_1, \epsilon\right), && \delta\left(q_0, \mathrm{b}, \mathrm{B}\right) = \left(q_1, \epsilon\right), \\
&\delta\left(q_1, \mathrm{b}, \mathrm{B}\right) = \left(q_1, \epsilon\right), && \delta\left(q_1, \mathrm{a}, \mathrm{A}\right) = \left(q_1, \epsilon\right), \\
&\delta\left(q_1, \epsilon, \mathrm{Z}\right) = \left(q_2, \epsilon\right)
\end{aligned}
$$

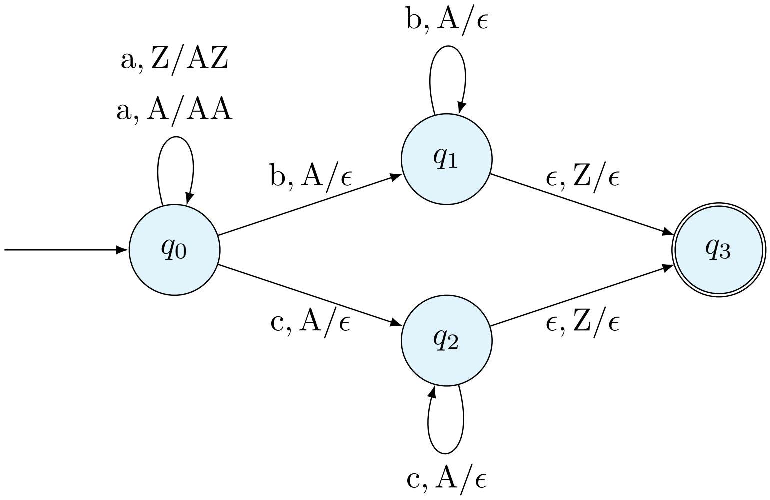
このとき、入力 ababbaba 及び babaabab に対する動作を示しなさい。

解答例

$$
\begin{aligned}
(q_0, \mathrm{ababbaba}, \mathrm{Z}) &\vdash (q_0, \mathrm{babbaba}, \mathrm{AZ}) \\
&\vdash (q_0, \mathrm{abbaba}, \mathrm{BAZ}) \\
&\vdash (q_0, \mathrm{bbaba}, \mathrm{ABAZ}) \\
&\vdash (q_0, \mathrm{baba}, \mathrm{BABAZ}) \\
&\vdash (q_1, \mathrm{aba}, \mathrm{ABAZ}) \\
&\vdash (q_1, \mathrm{ba}, \mathrm{BAZ}) \\
&\vdash (q_1, \mathrm{a}, \mathrm{AZ}) \\
&\vdash (q_1, \epsilon, \mathrm{Z}) \\
&\vdash (q_2, \epsilon, \epsilon)
\end{aligned}
$$

$$
\begin{aligned}
(q_0, \mathrm{babaabab}, \mathrm{Z}) &\vdash (q_0, \mathrm{abaabab}, \mathrm{BZ}) \\
&\vdash (q_0, \mathrm{baabab}, \mathrm{ABZ}) \\
&\vdash (q_0, \mathrm{aabab}, \mathrm{BABZ}) \\
&\vdash (q_0, \mathrm{abab}, \mathrm{ABABZ}) \\
&\vdash (q_1, \mathrm{bab}, \mathrm{BABZ}) \\
&\vdash (q_1, \mathrm{ab}, \mathrm{ABZ}) \\
&\vdash (q_1, \mathrm{b}, \mathrm{BZ}) \\
&\vdash (q_1, \epsilon, \mathrm{Z}) \\
&\vdash (q_2, \epsilon, \epsilon)
\end{aligned}
$$

課題 2　以下のような決定性有限オートマトン M を考える。



$$
\begin{aligned}
Q &= \{q_0.q_1, q_2, q_3\} \\
\Sigma &= \{\mathrm{a}, \mathrm{b}, \mathrm{c}\} \\
\Gamma &= \{\mathrm{A}, \mathrm{Z}\} \\
F &= \{q_3\}
\end{aligned}
$$

$$
\begin{aligned}
\delta\left(q_0, \mathrm{a}, \mathrm{Z}\right) &= \left(q_0, \mathrm{AZ}\right), & \delta\left(q_0, \mathrm{a}, \mathrm{A}\right) &= \left(q_0, \mathrm{AA}\right), \\
\delta\left(q_0, \mathrm{b}, \mathrm{A}\right) &= \left(q_1, \epsilon\right), & \delta\left(q_0, \mathrm{c}, \mathrm{A}\right) &= \left(q_2, \epsilon\right), \\
\delta\left(q_1, \mathrm{b}, \mathrm{A}\right) &= \left(q_1, \epsilon\right), & \delta\left(q_2, \mathrm{c}, \mathrm{A}\right) &= \left(q_2, \epsilon\right), \\
\delta\left(q_1, \epsilon, \mathrm{Z}\right) &= \left(q_3, \epsilon\right), & \delta\left(q_2, \epsilon, \mathrm{Z}\right) &= \left(q_3, \epsilon\right)
\end{aligned}
$$

このとき、入力 aaabbb 及び aacc に対する動作を示しなさい。

解答例

$$
\begin{aligned}
(q_0, \mathrm{aaabbb}, \mathrm{Z}) &\vdash (q_0, \mathrm{aabbb}, \mathrm{AZ}) \\
&\vdash (q_0, \mathrm{abbb}, \mathrm{AAZ}) \\
&\vdash (q_0, \mathrm{bbb}, \mathrm{AAAZ}) \\
&\vdash (q_1, \mathrm{bb}, \mathrm{AAZ}) \\
&\vdash (q_1, \mathrm{b}, \mathrm{AZ}) \\
&\vdash (q_1, \epsilon, \mathrm{Z}) \\
&\vdash (q_3, \epsilon, \epsilon)
\end{aligned}
$$

$$
\begin{aligned}
(q_0, \mathrm{aacc}, \mathrm{Z}) &\vdash (q_0, \mathrm{acc}, \mathrm{AZ}) \\
&\vdash (q_0, \mathrm{cc}, \mathrm{AAZ}) \\
&\vdash (q_2, \mathrm{c}, \mathrm{AZ}) \\
&\vdash (q_2, \epsilon, \mathrm{Z}) \\
&\vdash (q_3, \epsilon, \epsilon)
\end{aligned}
$$